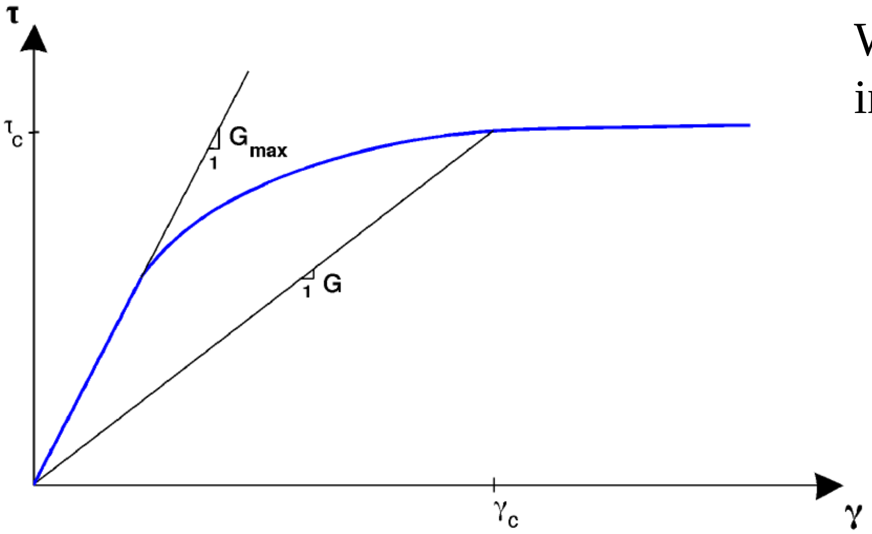
MASW – From theoretical considerations to practical consequences

Asoc. Prof. Jānis Karušs

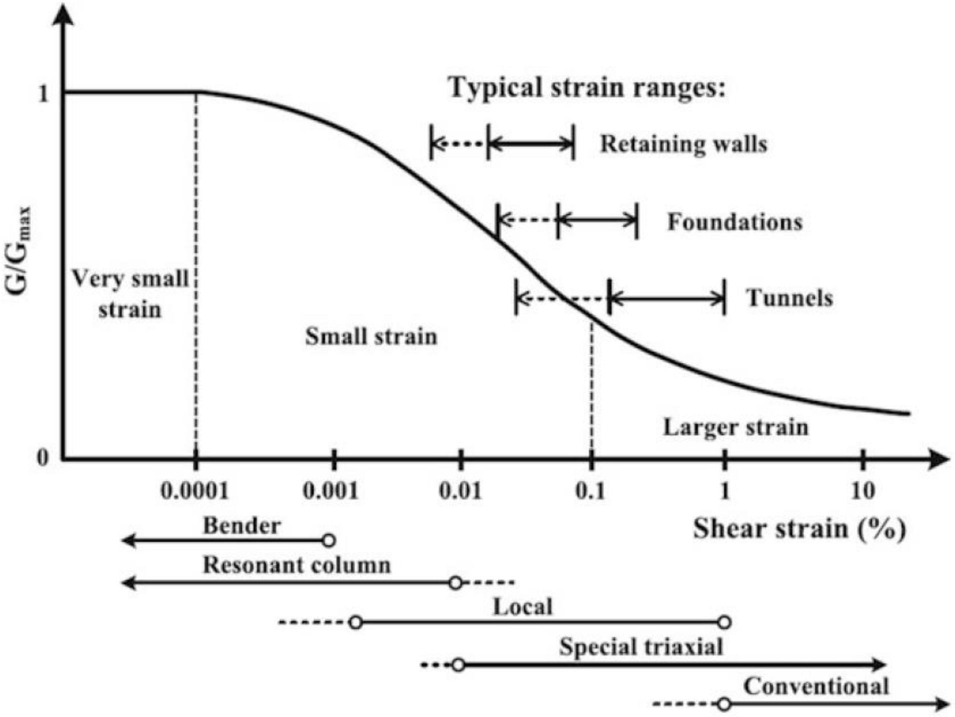
Link with mechanical properties



Why this is important for geotechnical investigation???

$$G_{max} = \rho V_S^2$$

G_{max} can be determined by using totally elastic deformation

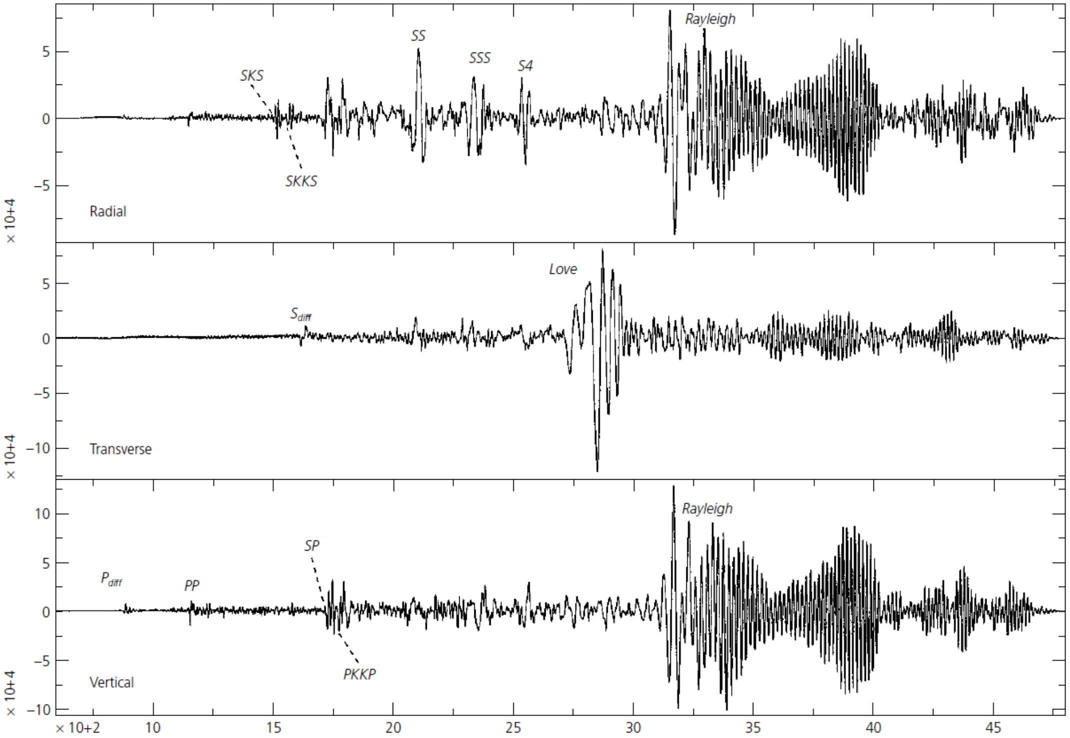


Surface waves

Already in the 19th century different type of waves were observed in seismic records after earthquakes.

Lord Reyleight in year 1885 published mathematical description of one type of those waves.

In year 1911 Love described another type that was later named after him.



Rayleigh, J.W.S., 1885. On waves propagated along the plane surface of an elastic solid. Proc. London Math. Soc. 17, 4–11.

Love, A.E.H., 1911. Some Problems of Geodynamics. Cambridge University Press.

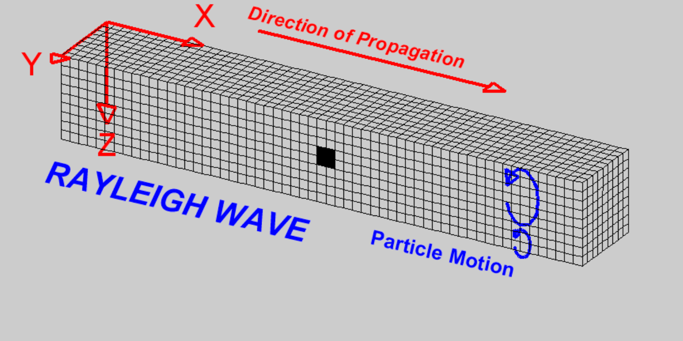
Surface waves

Usually, simple explanation and visualisation of surface waves are presented.

Movement along the surface:

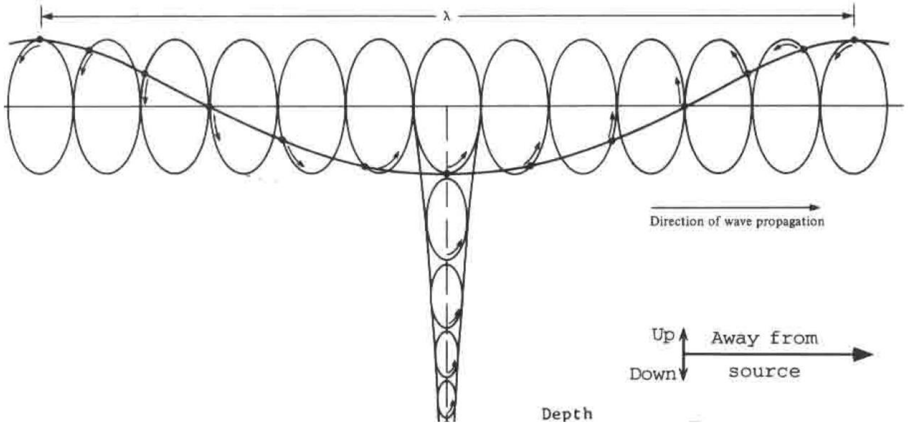
$$u = -0,423kA\sin k\left(x - V_R t\right)$$

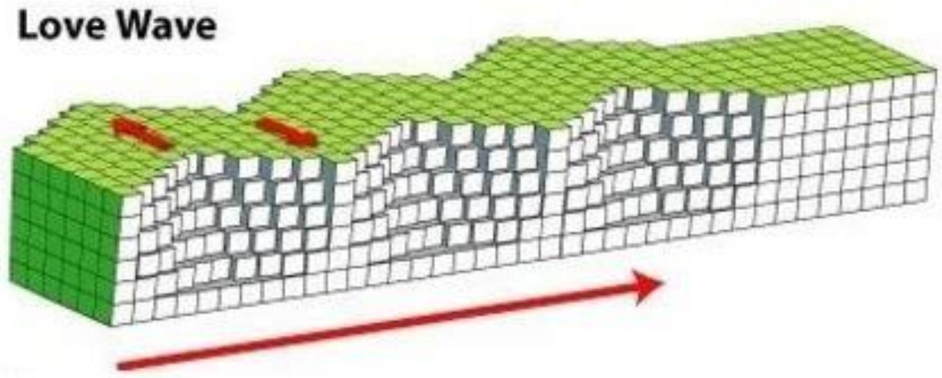
$$w = 0,620kA\cos k\left(x - V_R t\right)$$

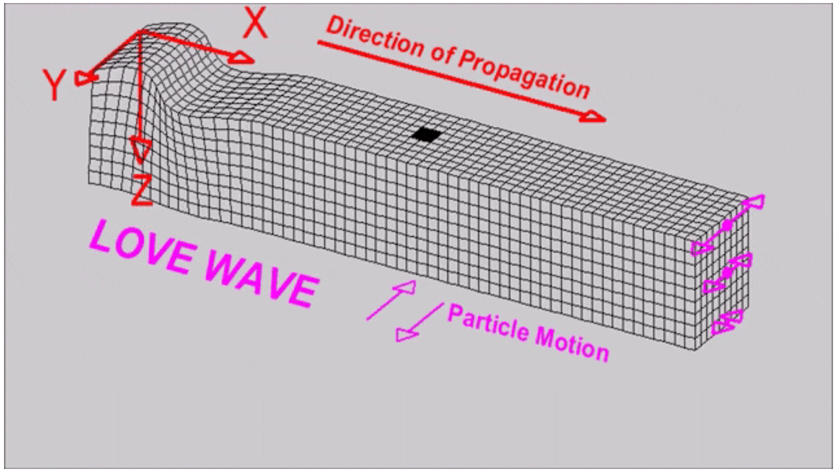


$$V_R = 0,919\beta$$

β – S wave propagation speed.



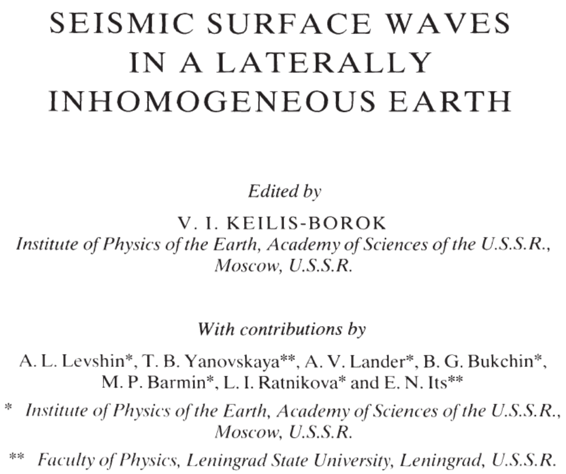




$$\beta_2 > V_L > \beta_1$$

Surface waves

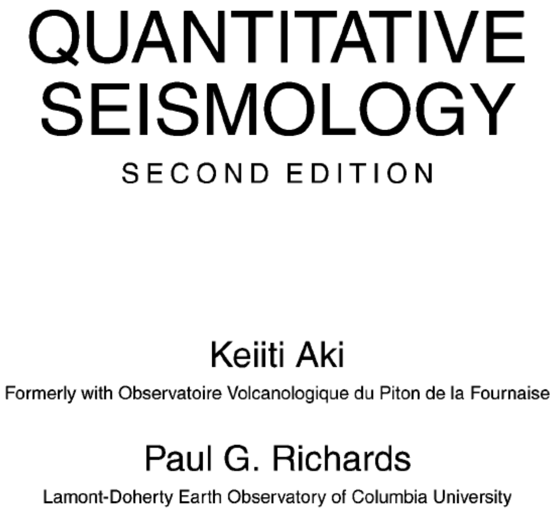
In every textbook about seismology there are derivation of mathematical description of surface waves.



SEISMIC SURFACE WAVES
IN A LATERALLY
INHOMOGENEOUS EARTH

Edited by
V. I. KEILIS-BOROK
Institute of Physics of the Earth, Academy of Sciences of the U.S.S.R., Moscow, U.S.S.R.

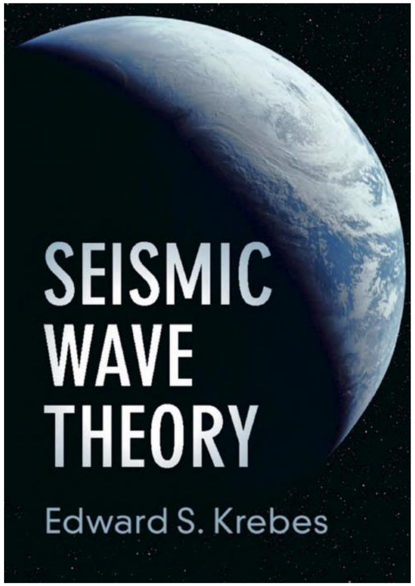
With contributions by
A. L. Levshin*, T. B. Yanovskaya**, A. V. Lander*, B. G. Bukchin*, M. P. Barmin*, L. I. Ratnikova* and E. N. Its**
* *Institute of Physics of the Earth, Academy of Sciences of the U.S.S.R., Moscow, U.S.S.R.*
** *Faculty of Physics, Leningrad State University, Leningrad, U.S.S.R.*



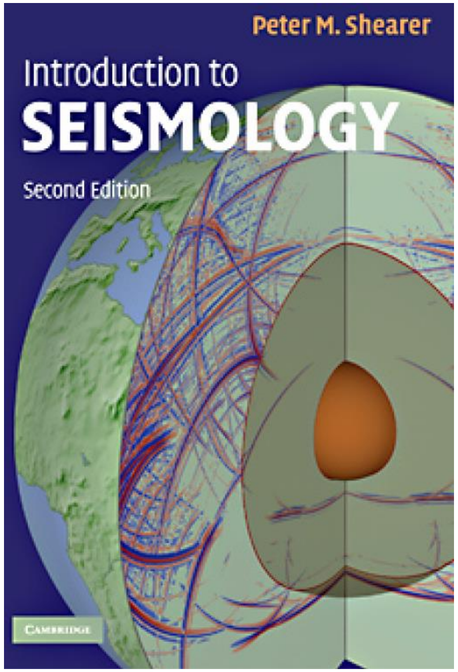
QUANTITATIVE
SEISMOLOGY
SECOND EDITION

Keiiti Aki
Formerly with Observatoire Volcanologique du Piton de la Fournaise

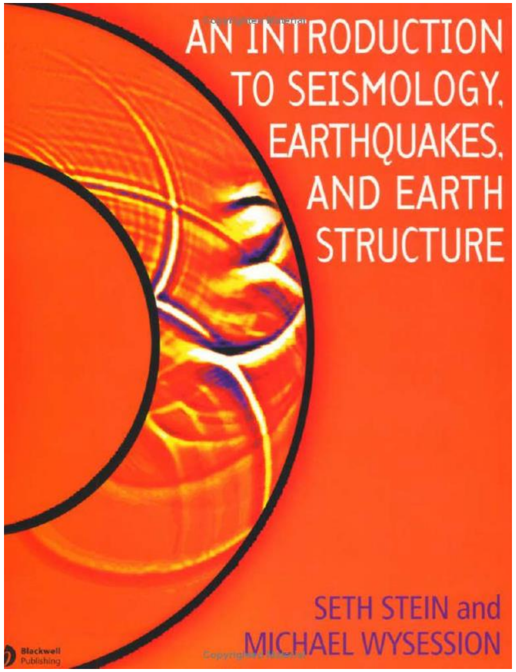
Paul G. Richards
Lamont-Doherty Earth Observatory of Columbia University



SEISMIC
WAVE
THEORY
Edward S. Krebes



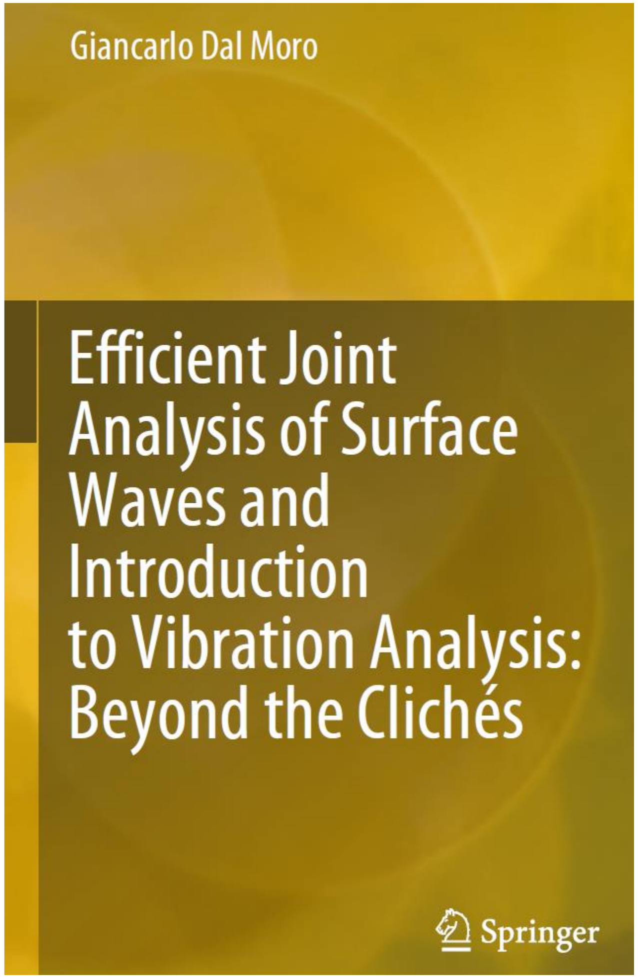
Peter M. Shearer
Introduction to
SEISMOLOGY
Second Edition
CAMBRIDGE



AN INTRODUCTION
TO SEISMOLOGY,
EARTHQUAKES,
AND EARTH
STRUCTURE
SETH STEIN and
MICHAEL WYSESSION
Blackwell Publishing

Surface waves

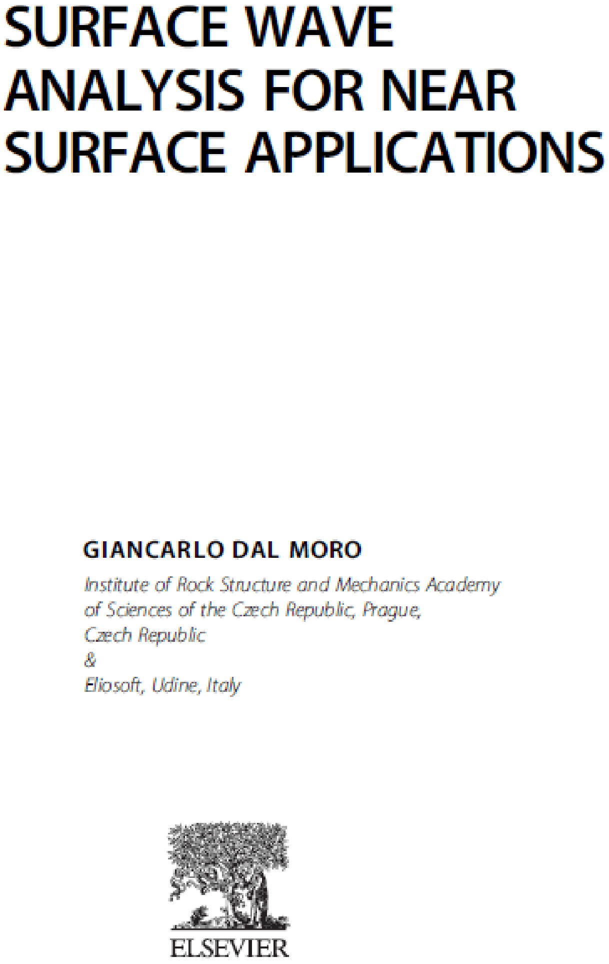
In textbooks dedicated to MASW usually it is not present



Giancarlo Dal Moro

Efficient Joint Analysis of Surface Waves and Introduction to Vibration Analysis: Beyond the Clichés

Springer



SURFACE WAVE ANALYSIS FOR NEAR SURFACE APPLICATIONS

GIANCARLO DAL MORO

Institute of Rock Structure and Mechanics Academy
of Sciences of the Czech Republic, Prague,
Czech Republic
&
Eliosoft, Udine, Italy

ELSEVIER

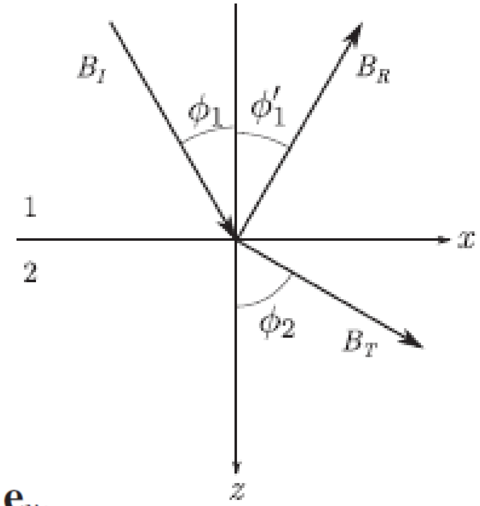
Surface waves

General concept behind derivation of a solution is to figure out hove we can describe surface waves using plane waves.

Evanescent waves are used.

Crytical angle

$$\phi_{1c} = \sin^{-1}(\beta_1/\beta_2).$$



If incidence angle is greater than critical angle, we get wave that travels in the second medium, parallel to the boundary and its amplitude decreases with depth exponentially.

Displacement of the transmitted wave:

$$\mathbf{u}^{(T)} = B_T \exp\left[i\omega(\mathbf{s}\cdot\mathbf{x} - t)\right]\mathbf{e}_y = B_T \exp\left[i\omega\left(px + \frac{\cos\phi_2}{\beta_2}z - t\right)\right]\mathbf{e}_y,$$

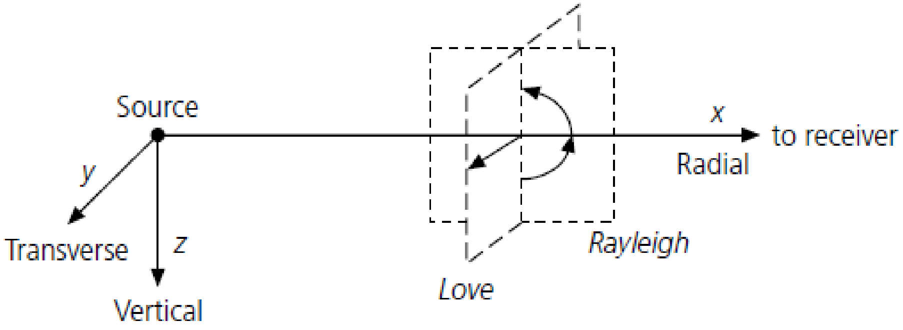
As sin function of refracted angle is imaginary:

$$\begin{aligned}\cos\phi_2 &= \sqrt{1-\sin^2\phi_2} = \pm i\sqrt{\sin^2\phi_2 - 1} \\ &= i\sqrt{\frac{{\beta_2}^2}{{\beta_1}^2}\sin^2\phi_1 - 1} = i\sqrt{{\beta_2}^2p^2 - 1} \qquad (\phi_1 \geq \phi_{1c}\,,\ \omega > 0).\end{aligned}$$

We get plane wave that travels in positive x direction with speed 1/p.

$$\mathbf{u}^{(T)} = B_T \exp\left[-\frac{\omega}{\beta_2}\sqrt{\beta_2^2p^2 - 1}\,z\right]\exp\left[i\omega(px - t)\right]\mathbf{e}_y \qquad (\phi_1 \geq \phi_{1c}\,,\ \omega > 0).$$

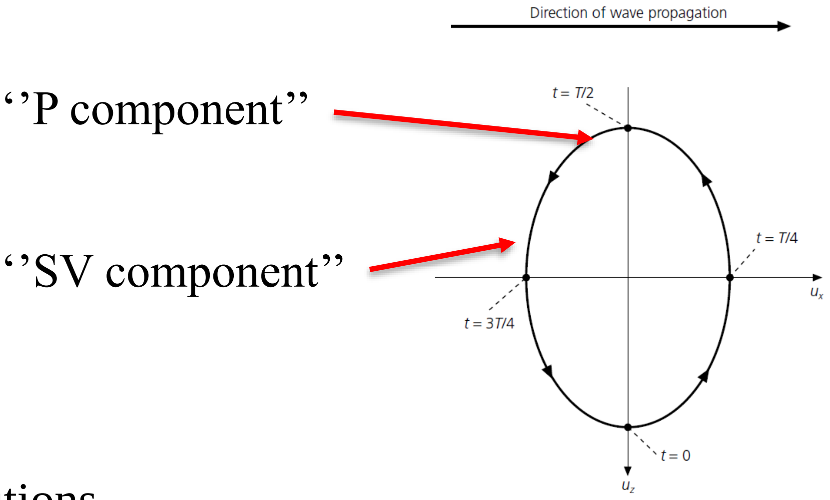
Rayleigh waves



Usually, it is stated that Rayleigh waves are combination of P and SV waves.

To derive mathematical solution for the problem, few boundary conditions are stated:

Free surface at the Earth surface,
Vanishing amplitudes at great depth.



‘’P component’’

‘’SV component’’

We start with P and SV wave potentials:

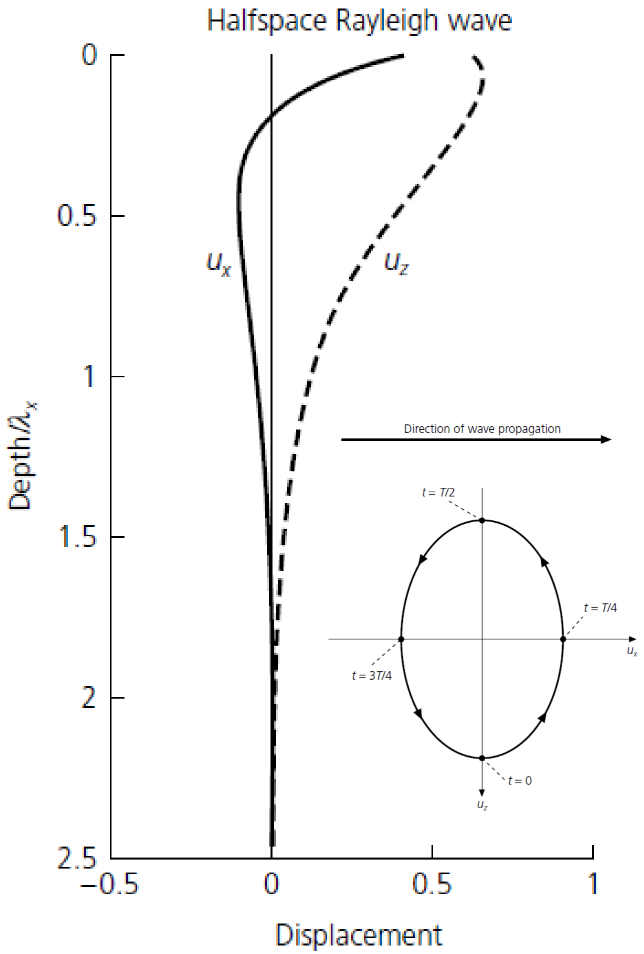
$$\phi = A \exp\left(i(\omega t - k_x x - k_x r_\alpha z)\right)$$

$$\psi = B \exp\left(i(\omega t - k_x x - k_x r_\beta z)\right)$$

We end up with cubic equation with 4 solutions.

Only one solution satisfy our boundary conditions: $c_x = (2 - 2/\sqrt{3})\beta = 0.92\ \beta$

Rayleigh waves



We can also derive the displacement.

$$u_x = Ak_x \sin(\omega t - k_x x)[\exp(-0.85\, k_x z) - 0.58 \exp(-0.39\, k_x z)],$$

$$u_z = Ak_x \cos(\omega t - k_x x)[-0.85 \exp(-0.85\, k_x z) + 1.47 \exp(-0.39\, k_x z)].$$

Notice that displacement is dependent of frequency!

Waves are more sensitive to mechanical properties of the space, where the displacement is the biggest.

As we have harmonic waves traveling only along the surface, it is reasonable to speak only about horizontal wavelength

$$\lambda_x = 2\pi / k_x$$

We derived mathematical description of Rayleigh waves over homogeneous half space. The one that we observe in seismograms is called ground roll, that essentially is Rayleigh waves over inhomogeneous half space.

Rayleigh waves

Penetration depth

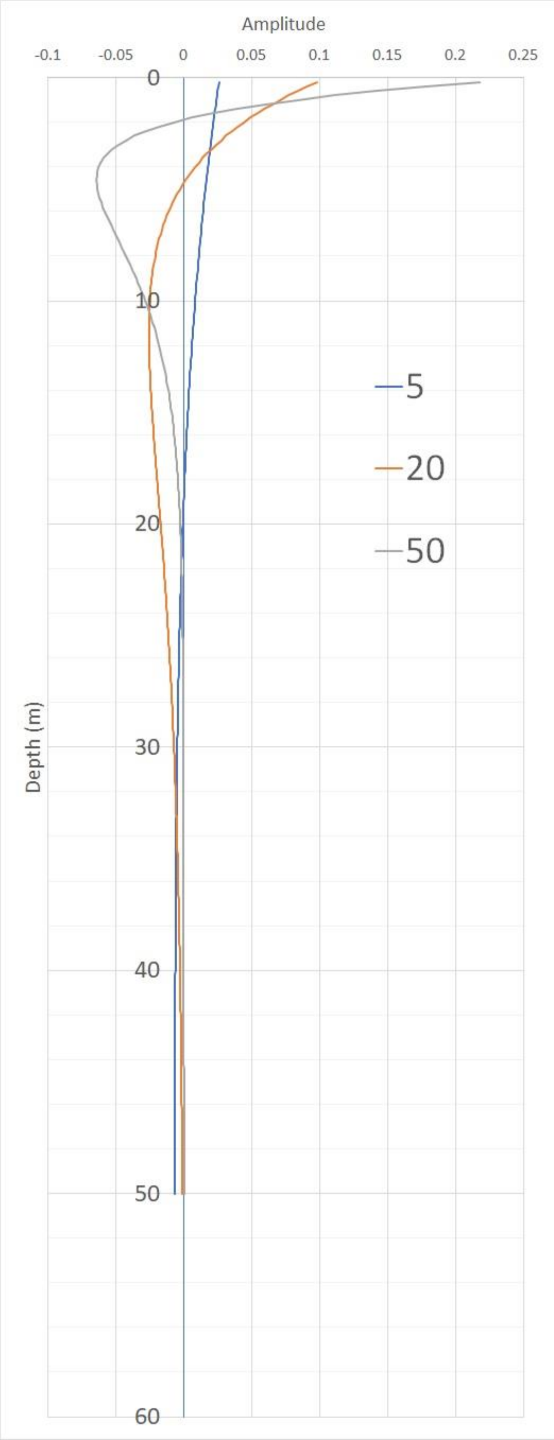
$$u_x = Ak_x \sin(\omega t - k_x x)[\exp(-0.85\, k_x z) - 0.58 \exp(-0.39\, k_x z)],$$

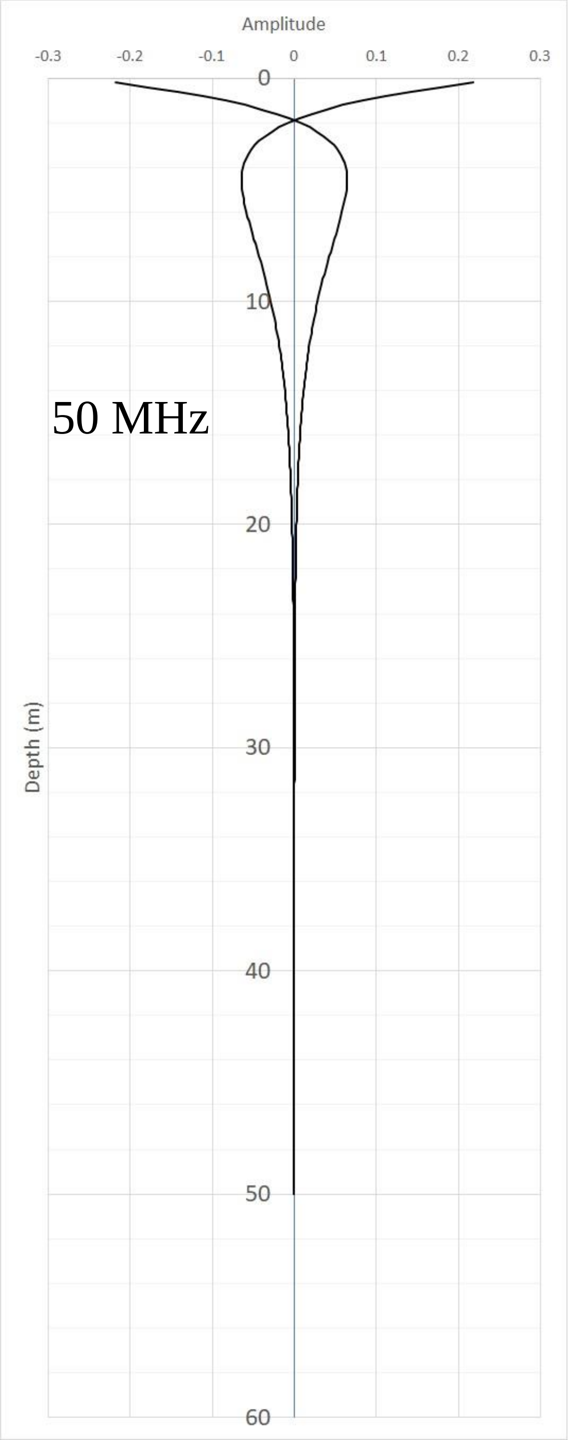
$$\lambda_x = 2\pi/\breve{k}_x$$

All frequencies are influenced also by shallow layers.

In this example propagation speed is 500 m/s

V=λ x f



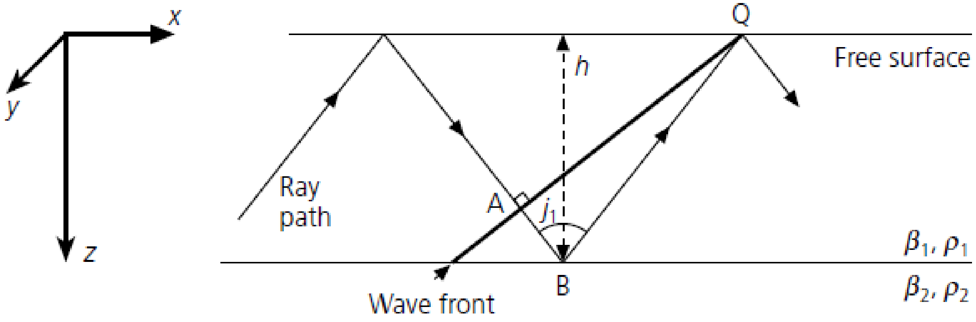


Love waves

Love waves are SH waves.

Love waves are more problematic in terms of mathematical derivation. It turns out that they can not exist in homogenious half space – vertical variation in S wave speed must be present.

Usually, solution with one layer that overlies half space is used.

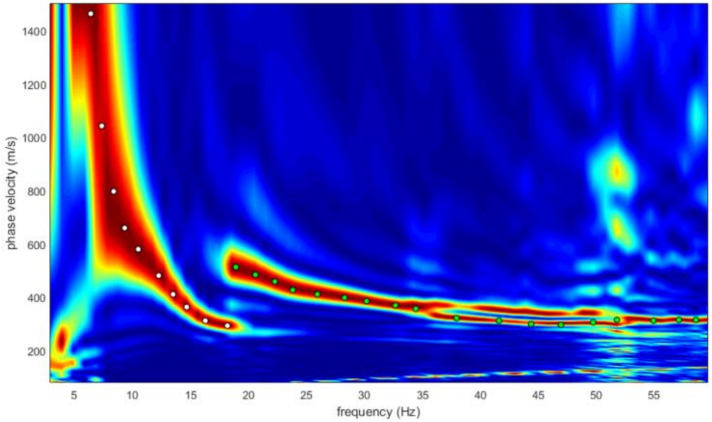


In the end we are looking for solutions that is formed via positive interference of upgoing and downgoing SH wave at the Earth surface.

$$u_y^-(x,z,t) = B_1 \exp\left(i(\omega t - k_x x - k_x r_{\beta_1} z)\right) + B_2 \exp\left(i(\omega t - k_x x + k_x r_{\beta_1} z)\right)$$

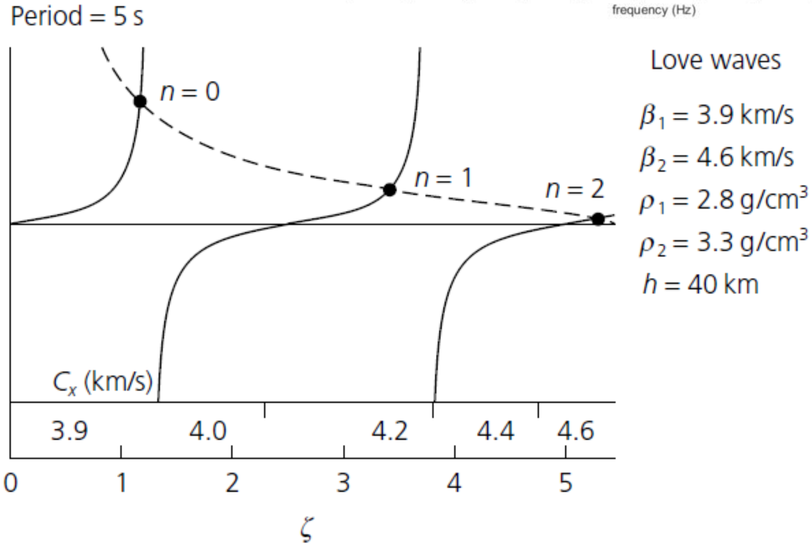
Love waves

We end up with solution that includes several frequencies. It is interpreted in a way that for constructive interference to be present there must be specific frequencies for specific horizontal apparent velocities.



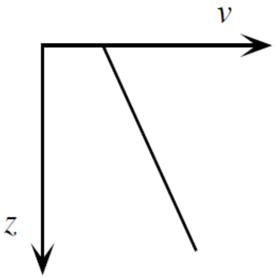
$$\tan(\omega\xi) = \left(\frac{\mu_2(1 - c_z^2/\beta_2^2)^{1/2}}{\mu_1}\right)\left(\frac{h}{c_z\xi}\right)$$

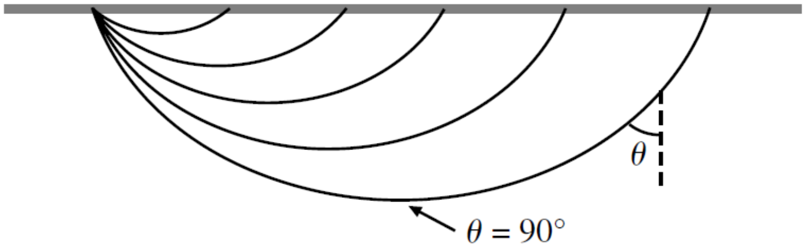
$$\zeta = (h/c_x)(c_x^2/\beta_1^2 - 1)^{1/2}$$



Sometimes Love waves are derived as a summ of S, SS etc. Multiples, but the same result is obtained

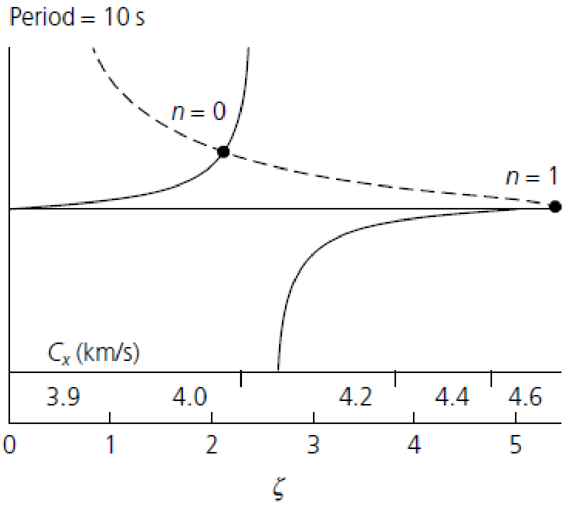
$$\omega = \frac{n2\pi + \pi/2}{T(p) - pX(p)} = \frac{n2\pi + \pi/2}{\tau(p)}$$

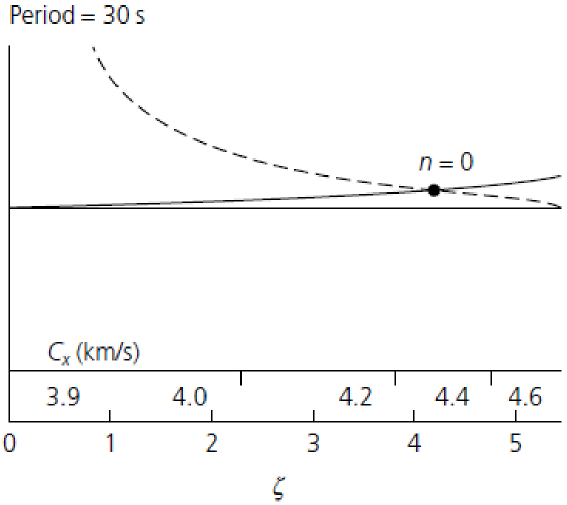


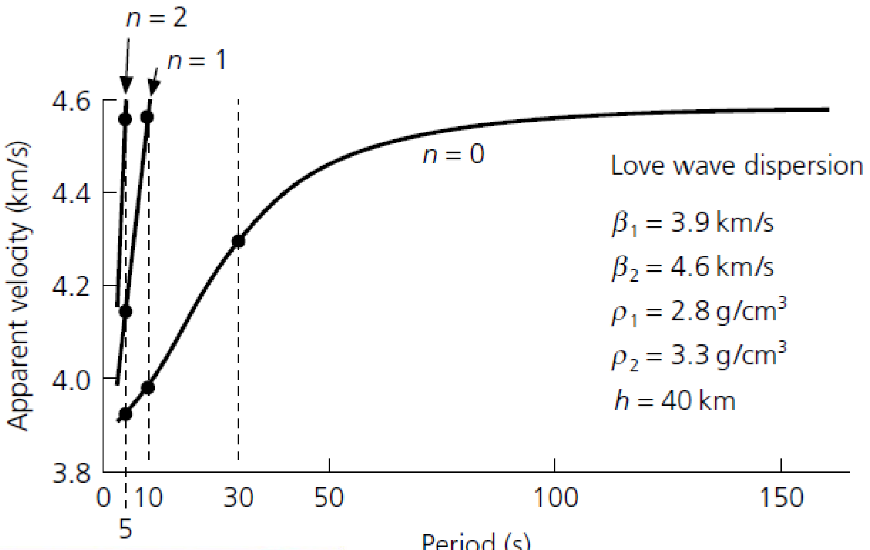


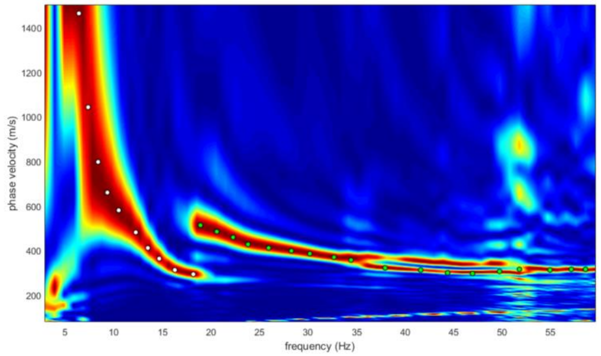
Love waves

We end up with typical display of frequency/apparent velocity









Notice that for lower frequencies only fundamental mode exist.

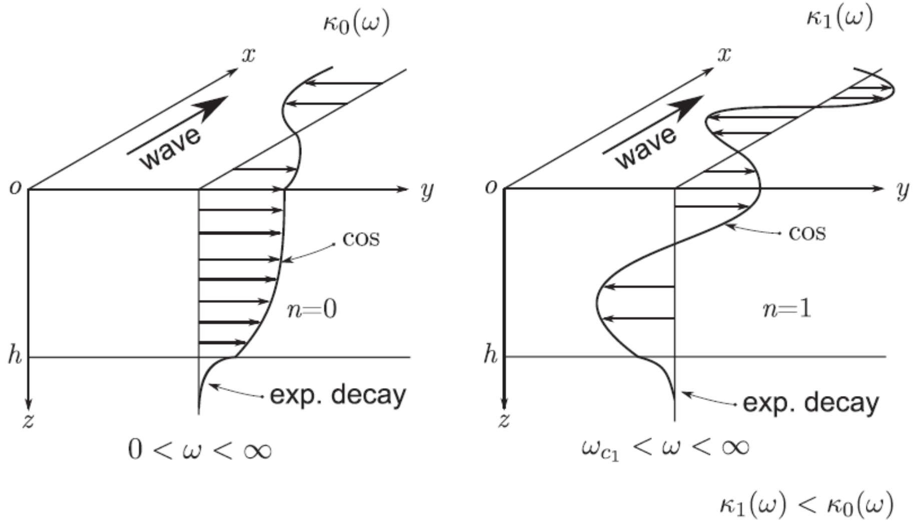
We may introduce the cutoff frequencie for the Nth mode.

$$\omega = \omega_{cn} = n\pi/[h(1/\beta_1^2 - 1/\beta_2^2)^{1/2}]$$

Notice that long period wave apparent speed approaches the speed of the halfspace.

Love waves

Displacement of the particles.

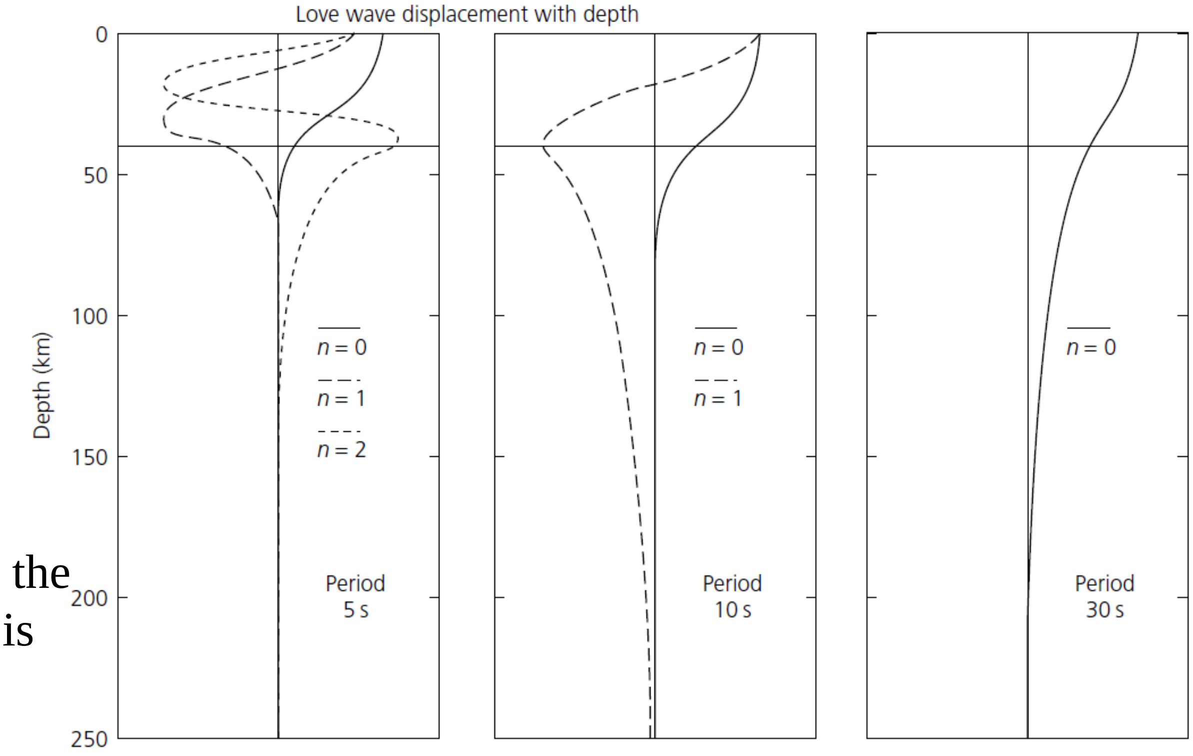


In the layer

$$u^{(1)} = 2A\cos\left[\omega\sqrt{\frac{1}{\beta_1^2}-\frac{1}{c_n^2}}\,z\right]\cos(\kappa x-\omega t).$$

In the halfspace

$$u^{(2)} = 2A\cos\left[\omega\sqrt{\frac{1}{\beta_1^2}-\frac{1}{c_n^2}}\,h\right]\exp\left[-\omega\sqrt{\frac{1}{c_n^2}-\frac{1}{\beta_2^2}}\,(z-h)\right]\cos(\kappa x-\omega t)$$



Waves are more sensitive to mechanical properties of the space, where the displacement is the biggest.

Higher modes are more sensitive to the upper part of the cross section

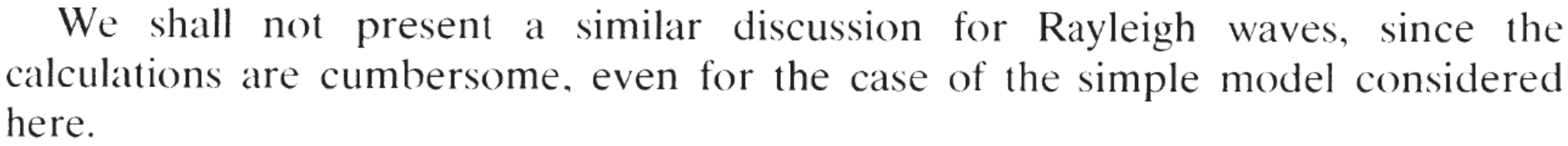
Love waves can be viewed as superposition of normal modes of the waveguide (our upper layer), this is reason why sometimes they are called guided waves.

Rayleigh waves

Previously we did not show the existence of many modes for Rayleigh waves.

Actually, mathematical proof of Rayleigh waves is seldom presented in literature about surface waves. In no one of previously mentioned books there is derivation of modes for Rayleigh waves.

Explanation form one of the books:

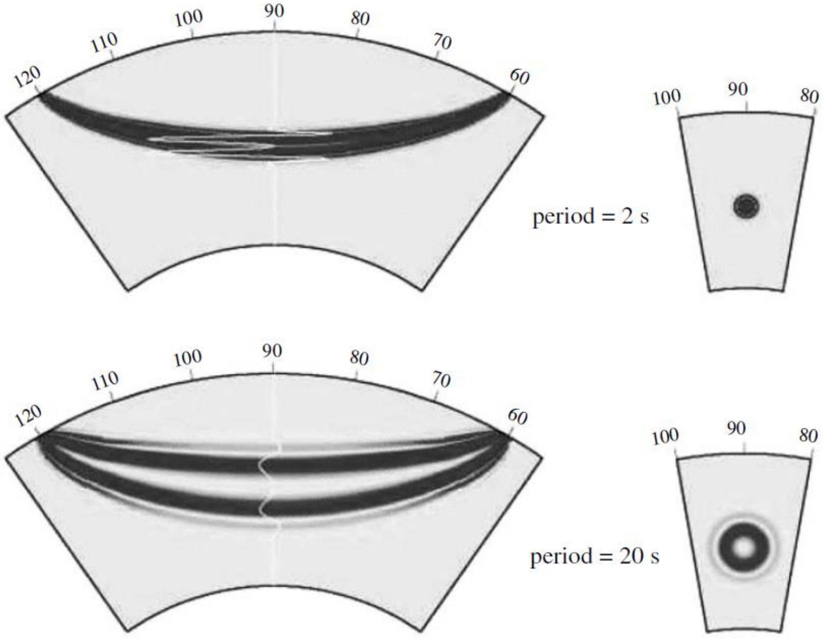


We shall not present a similar discussion for Rayleigh waves, since the calculations are cumbersome, even for the case of the simple model considered here.

Physical meaning of higher order modes is not intuitive and hard to imagine.

One must always remember that they exist even in the case with 2 layers, so it is not indicator of some complicated geological settings.

Travel path



Hove big is the area that influences observed waves?

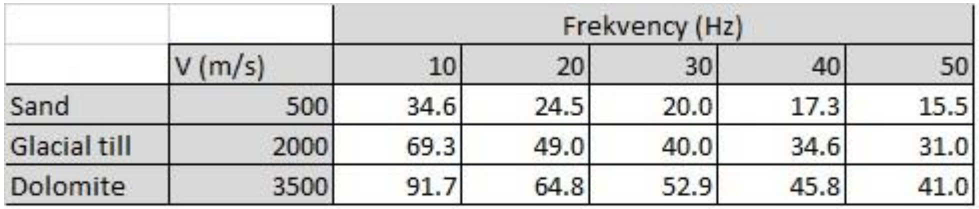
Usually there is discussion only about vertical direction.
Hove big area influences signal propagation in lateral direction for MASW studies is neglected at all.

We can use approach that is adopted for large scale seismic exploration «*banana-doughnut kernels*».

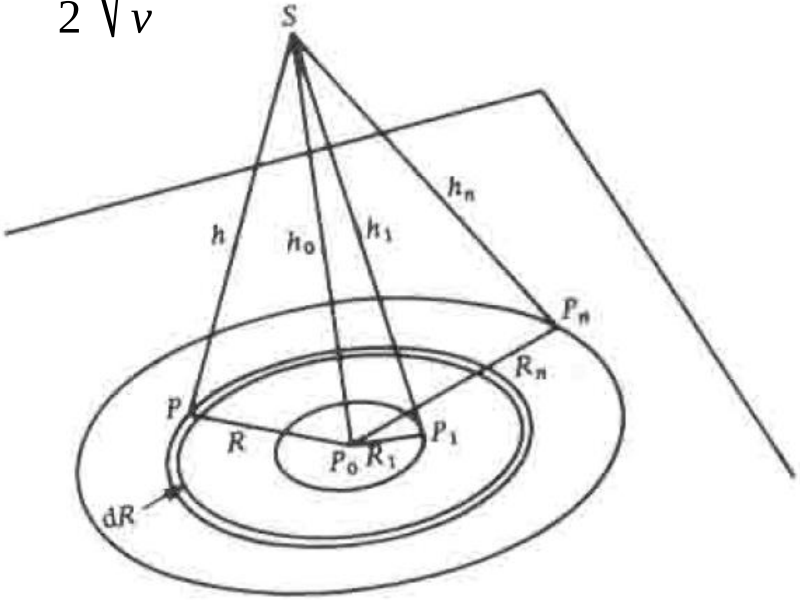
$$R_1 = \sqrt{\frac{\lambda h_0}{2}} = \frac{V}{2}\sqrt{\frac{t}{v}}$$

We do not use single ray but calculate the volume of influence by using First-Fresnel zone.

If 96m geophone spread is used.



		Frekvency (Hz)				
	V (m/s)	10	20	30	40	50
Sand	500	34.6	24.5	20.0	17.3	15.5
Glacial till	2000	69.3	49.0	40.0	34.6	31.0
Dolomite	3500	91.7	64.8	52.9	45.8	41.0



Field acquisition parameters

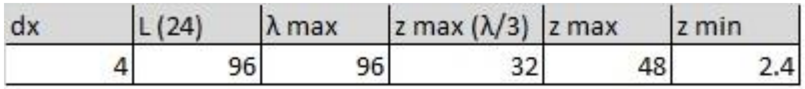
In most of the studies devoted to MASW similar suggestions are given:

$$\lambda_{max} \approx L$$

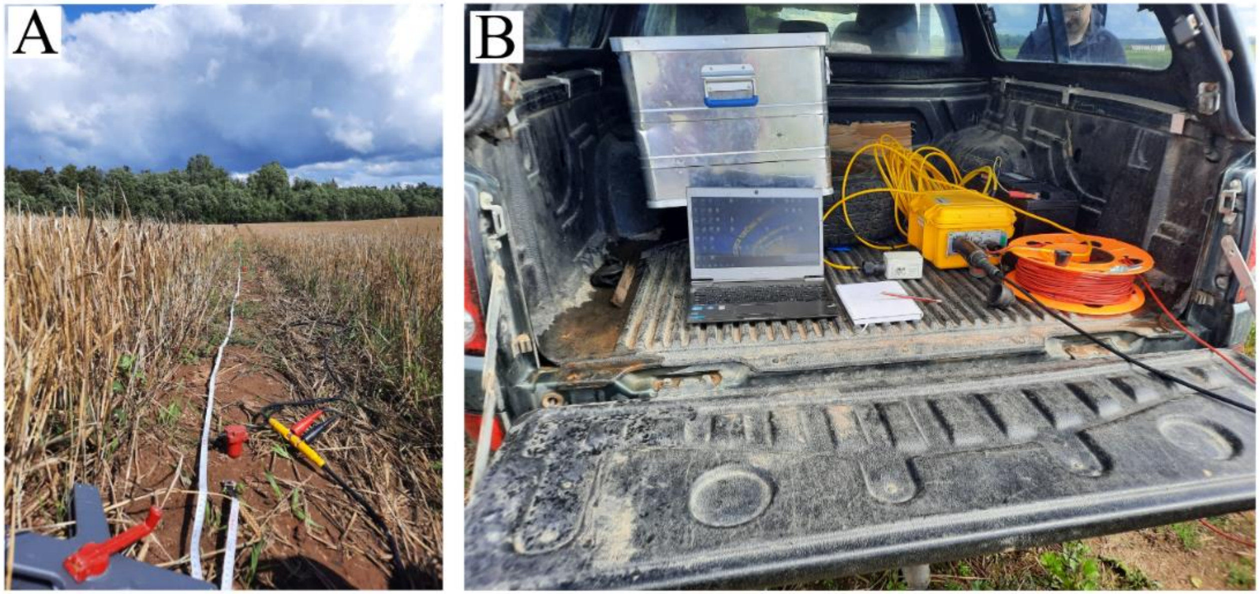
$$z_{max} \approx 0.5\lambda_{max}$$

more or less equal to $\lambda/3$

Standard survey parameters could be close to these or similar numbers:

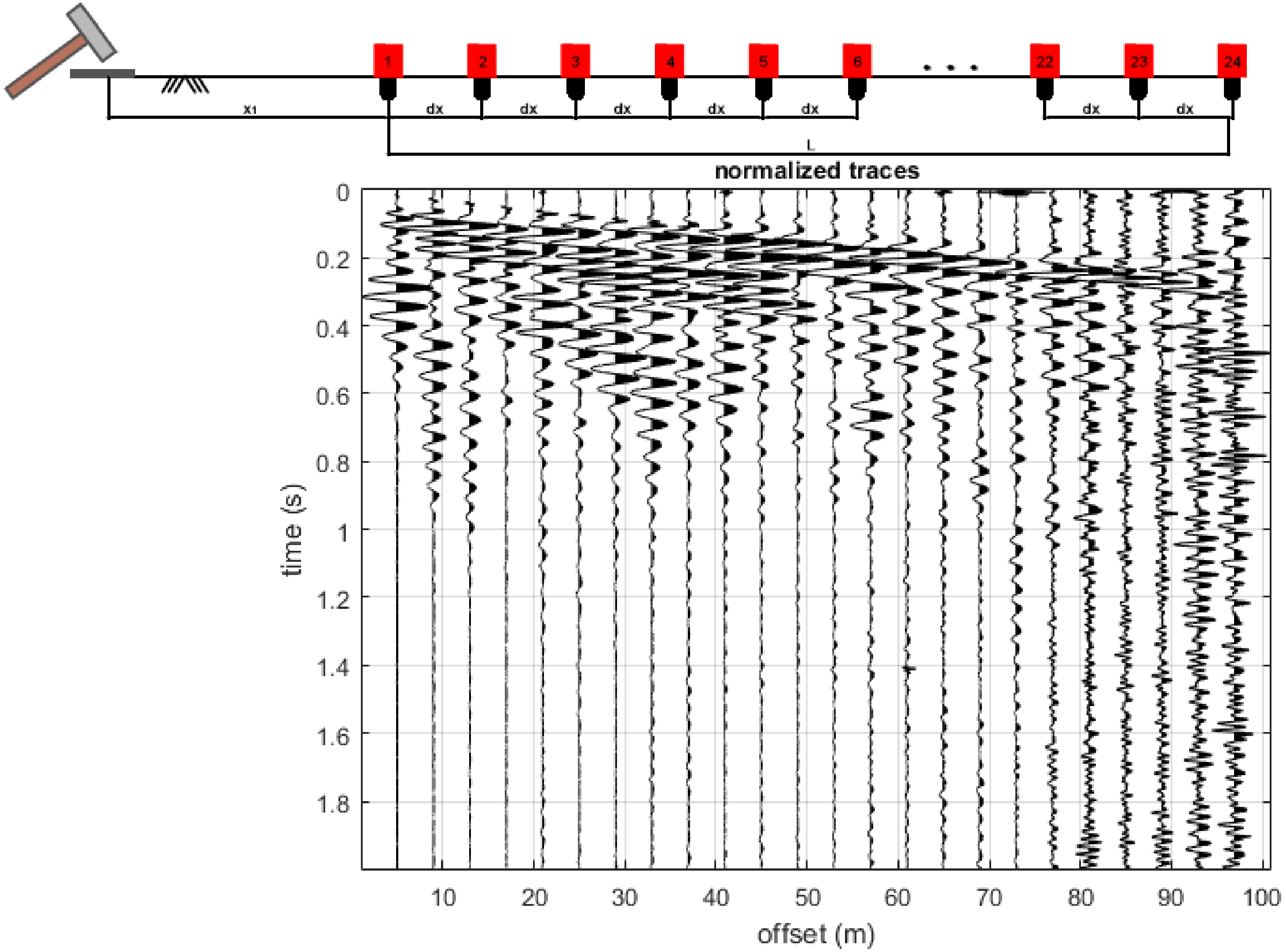


dx	L (24)	λ max	z max (λ/3)	z max	z min
4	96	96	32	48	2.4



Data processing

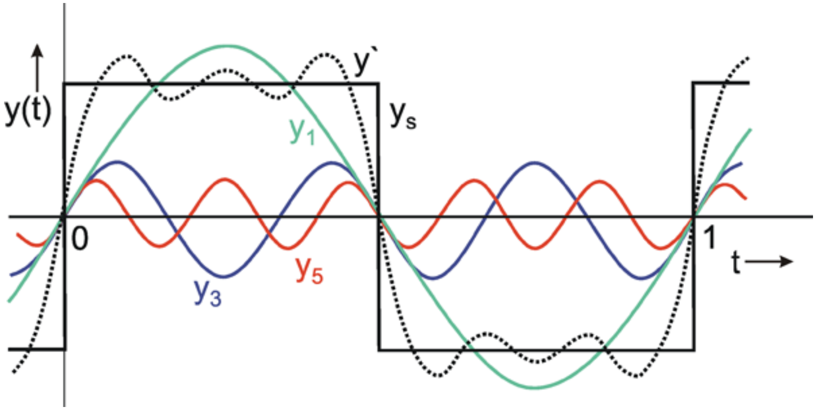
On the field a record of oscillations of each geophone was acquired.

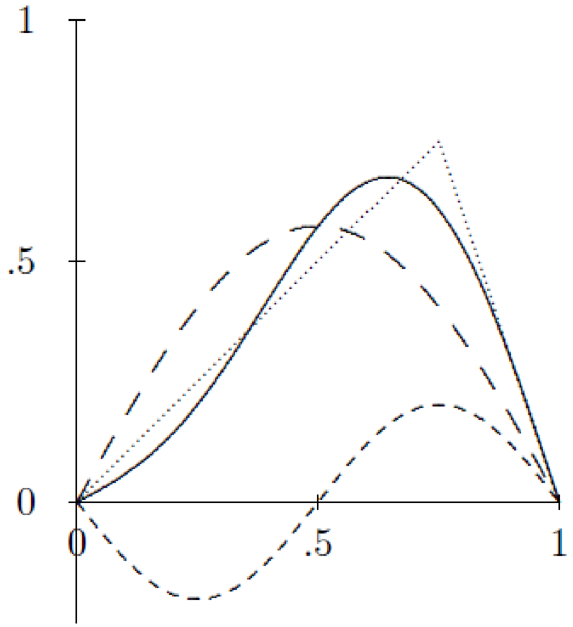


Frequency spectrum

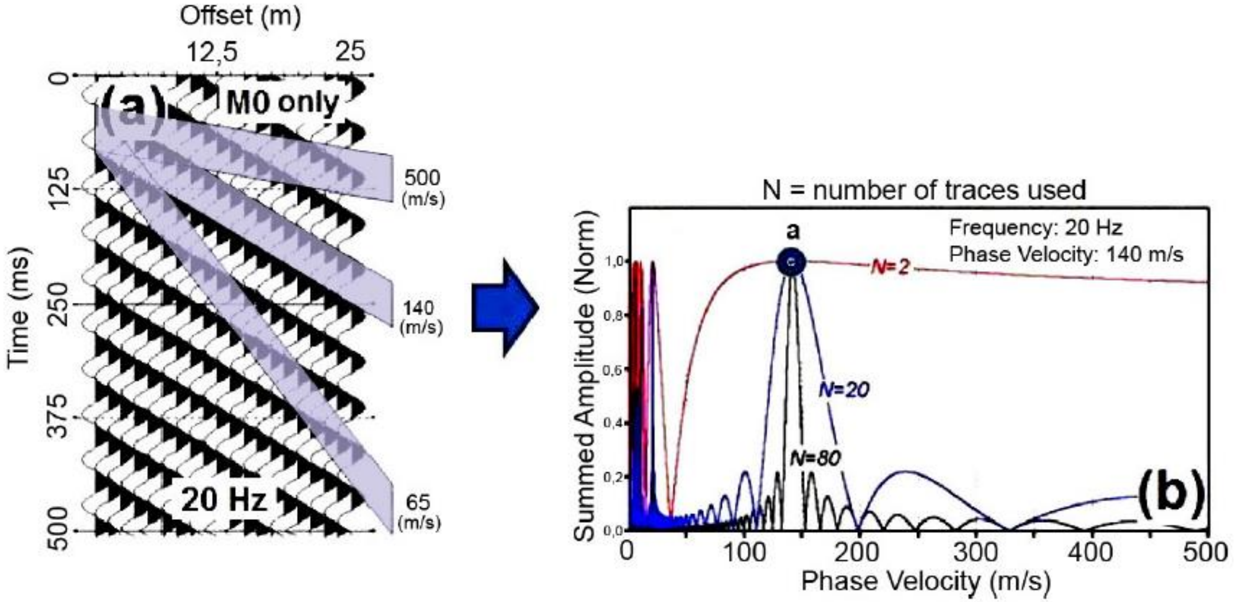
Fourier analysis – a way hove to describe our data.

$$\psi(x) = \sum_{n=1}^{N} c_n \sin n\pi x$$

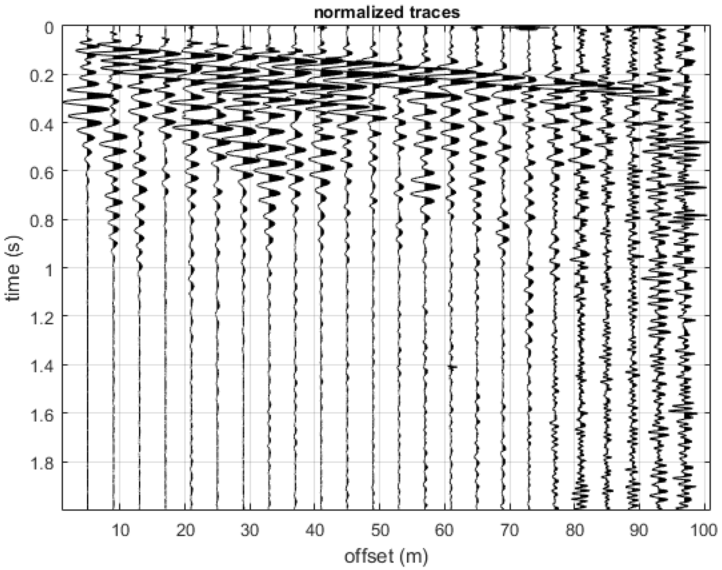




Back to MASW

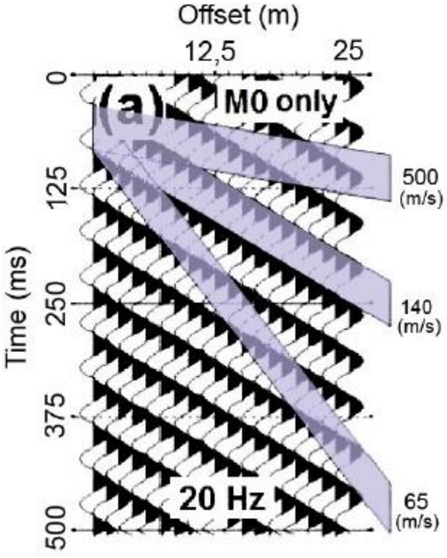


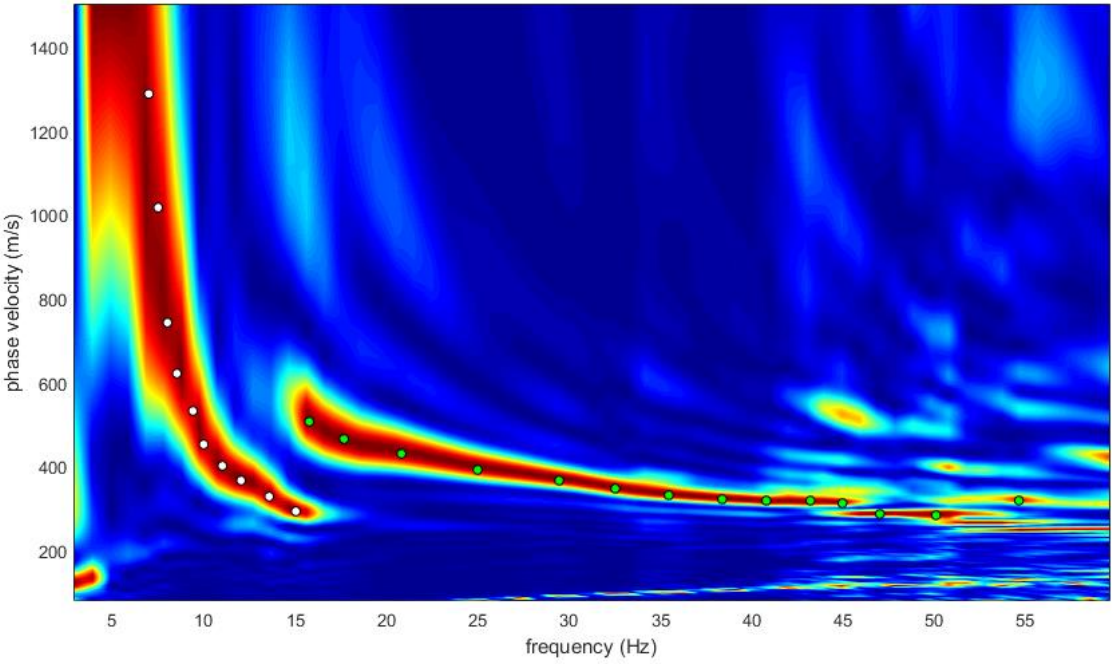
Frequency spectrum



Up until now everything was rather smooth.

You can obtain dataset without significant problems.

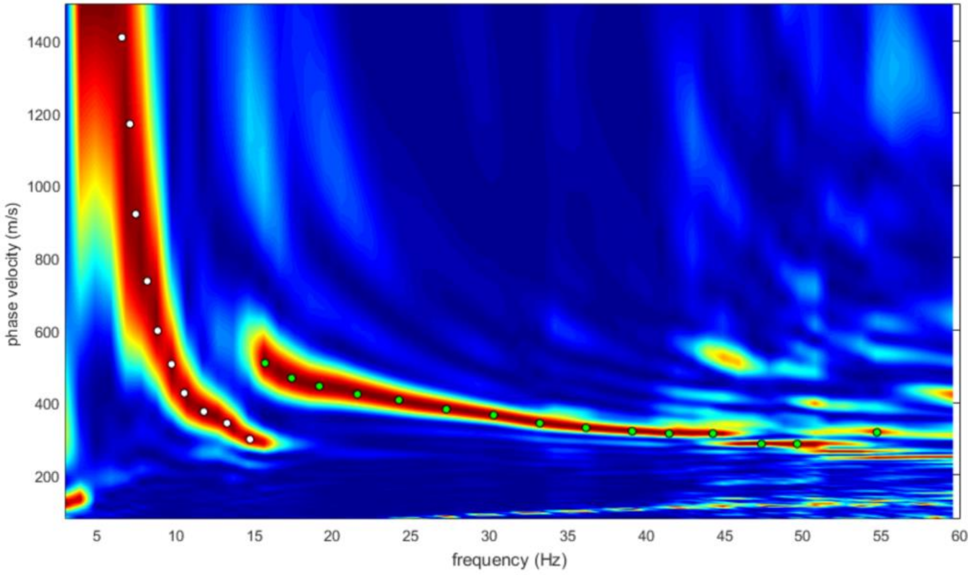


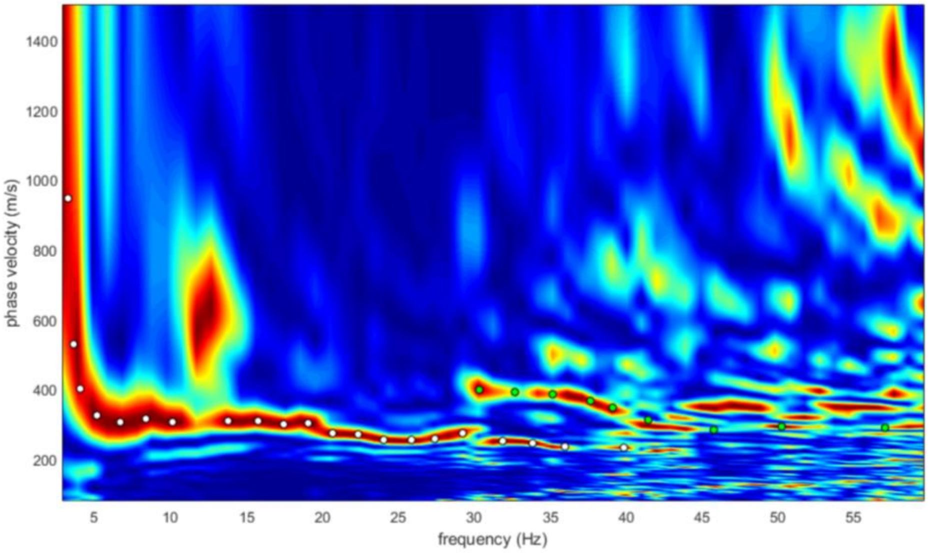


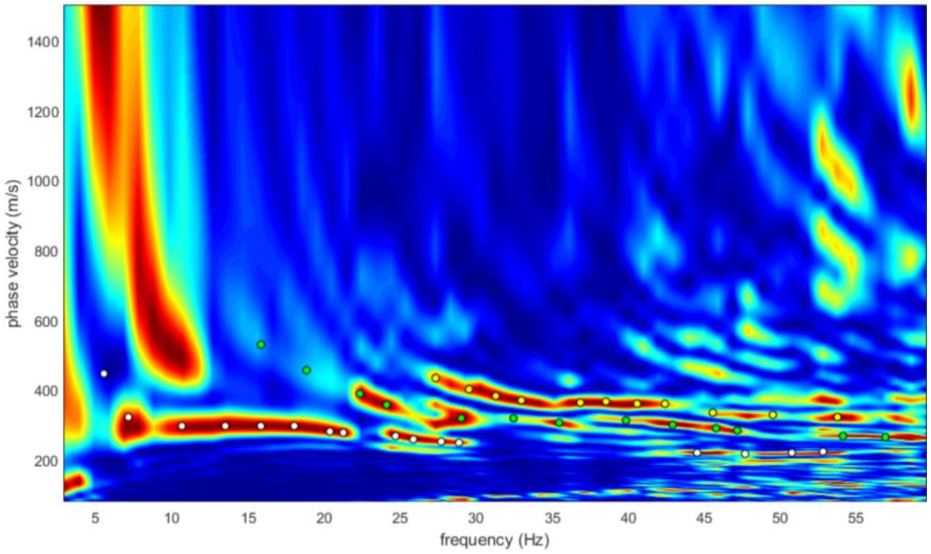
Mode picking

Sometimes it is rather complicated to identify different modes.

Currently it seems that rather weak layers make life more complicated.







Forward and inverse modelling

It is important to conduct **forward modelling first**.

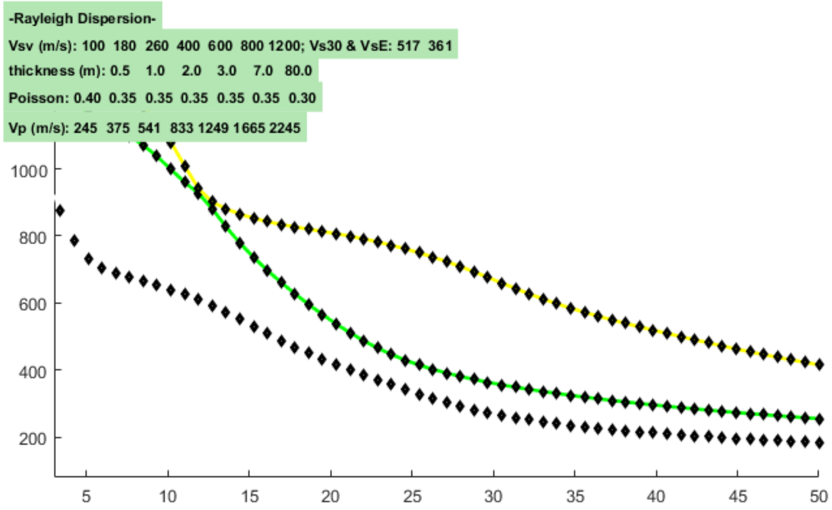
MASW survey without any geological information about the survey area is ambiguous.

Obtained results could be far from actual situation.

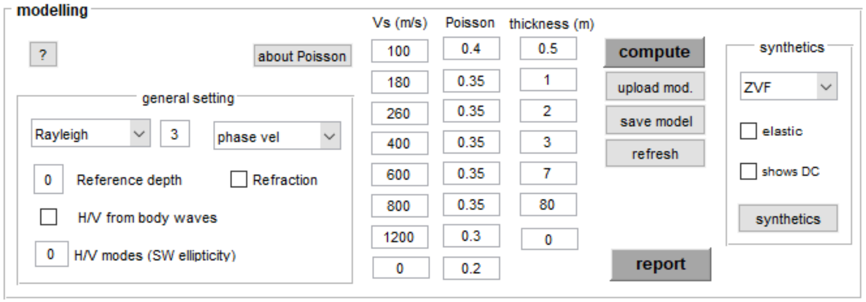
Inverse modelling:

To obtain final result, standard inverse modelling process is conducted – software creates a 1D model of the survey area that has the best fit to the field data.

Input parameters:
Layer count;
Layer thickness;
S wave speed interval;
Poisson ratio.



-Rayleigh Dispersion-
Vsv (m/s): 100 180 260 400 600 800 1200; Vs30 & VsE: 517 361
thickness (m): 0.5 1.0 2.0 3.0 7.0 80.0
Poisson: 0.40 0.35 0.35 0.35 0.35 0.35 0.30
Vp (m/s): 245 375 541 833 1249 1665 2245



modelling

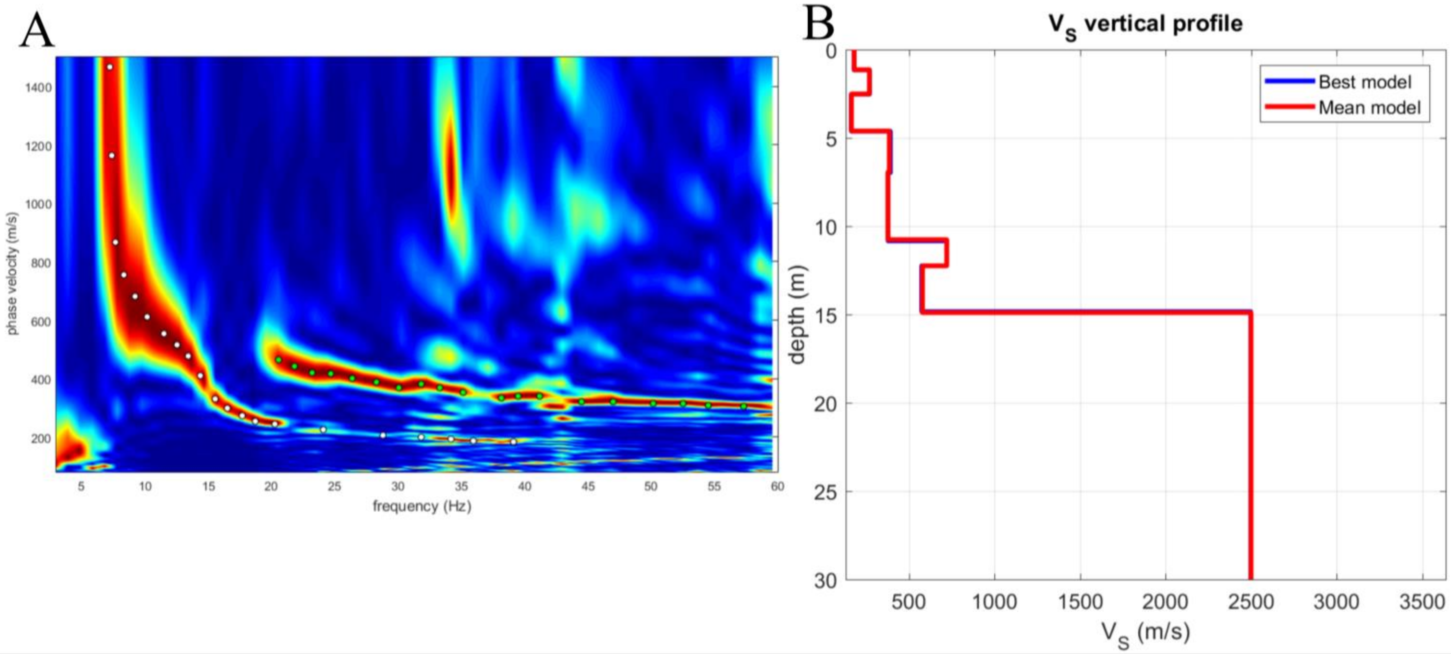
general setting
Rayleigh 3 phase vel
0 Reference depth ☐ Refraction
☐ H/V from body waves
0 H/V modes (SW ellipticity)

Vs (m/s)	Poisson	thickness (m)
100	0.4	0.5
180	0.35	1
260	0.35	2
400	0.35	3
600	0.35	7
800	0.35	80
1200	0.3	0
0	0.2	

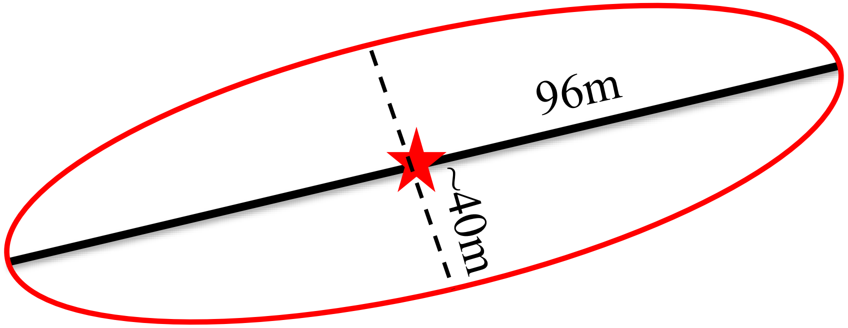
synthetics
ZVF
☐ elastic
☐ shows DC

Obtained result

Obtained result is a 1D model



It is assumed that in this area there are no lateral variations!!!



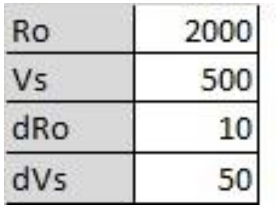
30 Hz

Maybe this is advantage and not a problem?

Precision of the obtained result

$$G_{max} = \rho V_S^2$$

$$R = \sqrt{\left(V_s{}^2 \Delta\rho\right)^2 + \left(2V_s \rho \Delta V_s\right)^2}$$



Ro	2000
Vs	500
dRo	10
dVs	50

If you assume such numbers partial error is around 10% for calculated G_{max}

In the end also error for G is important!!!

G_{sec}/G_{max} (often written as G/G_{max})

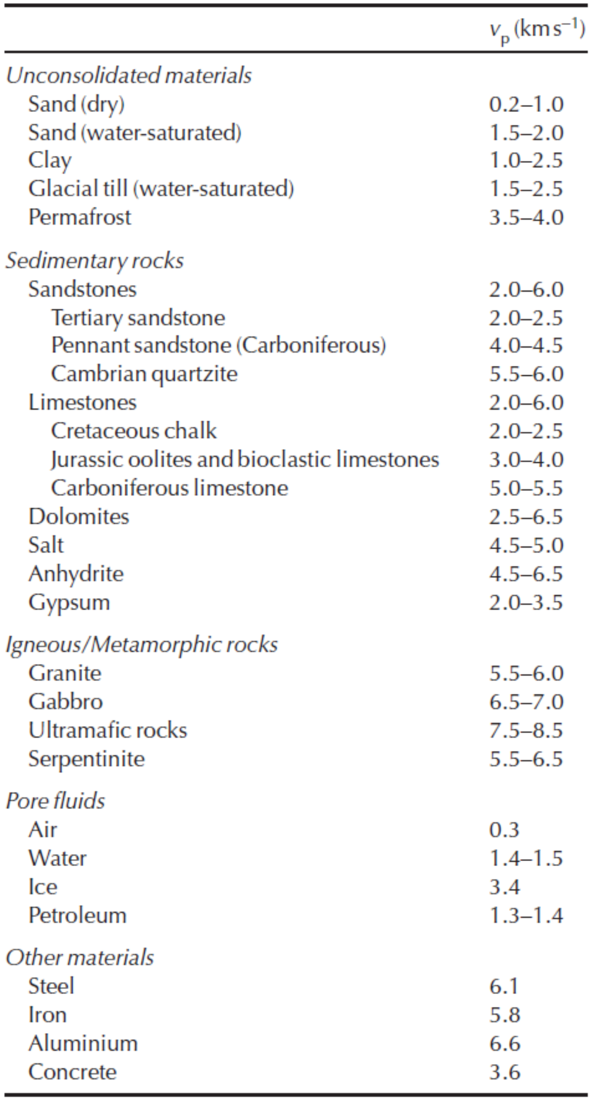
$$\left(\left(\frac{1}{\rho Vs^2}\Delta G\right)^2 + \left(\frac{G}{\rho^2 Vs^2}\Delta\rho\right)^2 + \left(\frac{2G}{\rho Vs^3}\Delta Vs\right)^2\right)$$

Problems to be solved in future

Database of typical S and P wave speed for different rock types.

More local data sets.

Joint analysis of Rayleigh and Love waves



	v_p (km s^{-1})
Unconsolidated materials	
Sand (dry)	0.2–1.0
Sand (water-saturated)	1.5–2.0
Clay	1.0–2.5
Glacial till (water-saturated)	1.5–2.5
Permafrost	3.5–4.0
Sedimentary rocks	
Sandstones	2.0–6.0
Tertiary sandstone	2.0–2.5
Pennant sandstone (Carboniferous)	4.0–4.5
Cambrian quartzite	5.5–6.0
Limestones	2.0–6.0
Cretaceous chalk	2.0–2.5
Jurassic oolites and bioclastic limestones	3.0–4.0
Carboniferous limestone	5.0–5.5
Dolomites	2.5–6.5
Salt	4.5–5.0
Anhydrite	4.5–6.5
Gypsum	2.0–3.5
Igneous/Metamorphic rocks	
Granite	5.5–6.0
Gabbro	6.5–7.0
Ultramafic rocks	7.5–8.5
Serpentinite	5.5–6.5
Pore fluids	
Air	0.3
Water	1.4–1.5
Ice	3.4
Petroleum	1.3–1.4
Other materials	
Steel	6.1
Iron	5.8
Aluminium	6.6
Concrete	3.6

Thank You for Your attention!